

Applied Statistics (Chapter 6)

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Chapter 6

Section 6.2

```
# Define 6 × 6 matrix P
P=matrix(c(.5,.5,0,0,0,0,.25,.5,.25,0,0,0,
0,.25,.5,.25,0,0,0,0,.25,.5,.25,0,
0,0,0,.25,.5,.25,0,0,0,0,.5,.5),
nrow=6,ncol=6,byrow=TRUE)
# Print P
P
# We generate  $X_n, n = 1, \dots, 50,000$  and save in s
# s is a  $50,000 \times 1$  vector with zeros
s=array(0,c(50000,1))
# Initialize  $X_1 = 3$  (s[1]=3)
s[1]=3
```

Section 6.2

```
# Generate  $X_n$   $n = 2, \dots, 50,000$  given  $X_{n-1}$ 
for (j in 2:50000)
s[j]=sample(1:6,size=1,prob=P[s[j-1],])
# Compute relative frequencies of each state
# using  $X_1, \dots, X_m$ ,  $m = 500, 200, 8000, 50000$ 
m=c(500,2000,8000,50000)
for (i in 1:4)
print(table(s[1:m[i]])/m[i])
# Define a true stationary distribution vector  $w$ 
w=matrix(c(.1,.2,.2,.2,.2,.1),nrow=1,ncol=6)
# Compute  $wP$  and see  $wP = w$ 
w%*%P
```

Section 6.7, 6.8 using WinBUGS

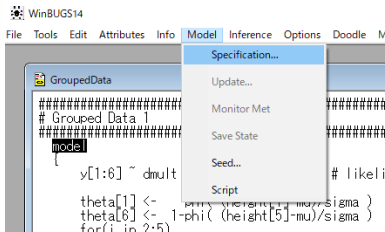
```
# First we specify the statistical model
model
{
#  $\mathbf{y} = (y_1, \dots, y_6)' \sim \text{Multinomial}(n, \boldsymbol{\theta})$ ,  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_6)'$ 
y[1:6] ~ dmulti(theta[1:6], n) # likelihood
#  $\theta_1 = \Phi\left(\frac{\text{height}_1 - \mu}{\sigma}\right)$ ,  $\theta_6 = 1 - \Phi\left(\frac{\text{height}_5 - \mu}{\sigma}\right)$ 
theta[1] <- phi( (height[1]-mu)/sigma )
theta[6] <- 1-phi( (height[5]-mu)/sigma )
#  $\theta_i = \Phi\left(\frac{\text{height}_i - \mu}{\sigma}\right) - \Phi\left(\frac{\text{height}_{i-1} - \mu}{\sigma}\right)$ ,  $i = 2, \dots, 5$ 
for(i in 2:5)
{
theta[i]<- phi( (height[i]-mu)/sigma ) - phi(
(height[i-1]-mu)/sigma )
}
}
```

Section 6.7, 6.8 using WinBUGS

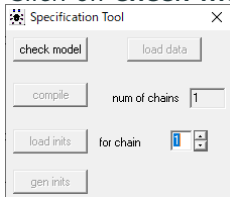
```
# Prior:  $\mu \sim N(0, 10^6)$ 
mu ~ dnorm(0, 0.000001)
# Prior:  $\log \sigma \sim N(0, 10^6)$ 
lsigma ~ dnorm(0, 0.000001)
# Prior:  $\sigma = \exp(\log \sigma)$ 
sigma <- exp(lsigma)
# the end of the model statement
}
# define variables for the dataset
list( n = 211, y = c(14, 30, 49, 70, 33, 15),
      height = c(66, 68, 70, 72, 74))
# set initial values for MCMC
list( mu = 70, lsigma=0 )
```

Introduction to WinBUGS

1. Highlight the **model** statement
2. **Model** menu → **Specification...**

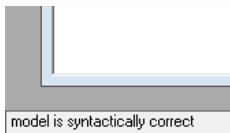


3. Click on **check model**

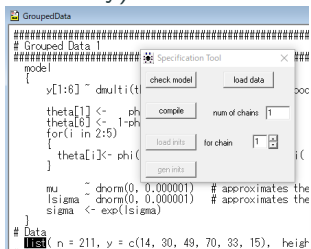


Introduction to WinBUGS

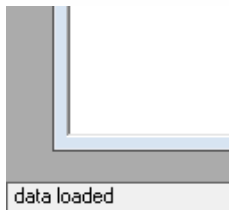
4. Check the message at the bottom



5. Highlight **list** and click on **load data** (repeat this step if necessary)

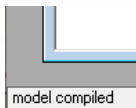


```
#####
# Grouped Data |
#####
model
{
  y[1:6] ~ dmulti(t
theta[1] <- ph
theta[6] <- 1-ph
for(i in 2:5)
{
  theta[i] <- phi(
}
mu ~ dnorm(0, 0.000001) # approximates the
lsigma ~ dnorm(0, 0.000001) # approximates the
sigma <- exp(lsigma)
}
# Data
.. list( n = 211, y = c(14, 30, 49, 70, 33, 15), heigh
```



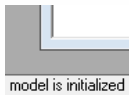
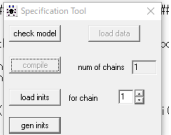
Introduction to WinBUGS

- Click on **compile** and check the message at the bottom

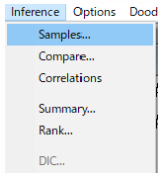


- Highlight **list** and click on **load inits** to set initial values of MCMC (or click on **gen inits** to generate initial values)

```
GroupedData
#####
# Grouped Data 1
#####
model
{
  y[1:6] ~ dmulti(t
  theta[1] <- phi
  theta[6] <- 1-phi
  for(i in 2:5)
  {
    theta[i] <- phi(
  }
  mu ~ dnorm(0, 0.000001) # approximates th
  lsigma ~ dnorm(0, 0.000001) # approximates th
  sigma <- exp(lsigma)
}
# Data
list( n = 211, y = c(14, 30, 49, 70, 33, 15), heig
# Initial values
list( mu = 70, lsigma=0 )
#####
```



8. Inference menu → Samples...

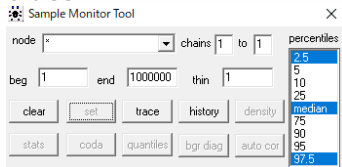
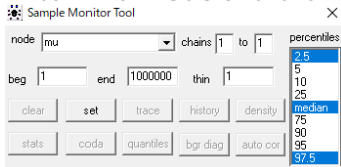


9. Enter **mu** for **node** and click on **set**

10. Enter **lsigma** for **node** and click on **set**

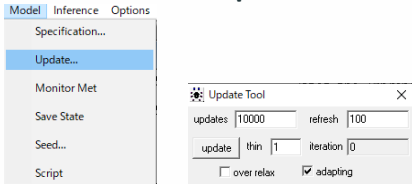
11. Enter **sigma** for **node** and click on **set**

12. Enter ***** for **node** and click on **trace**

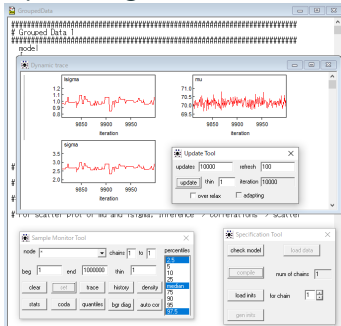


Introduction to WinBUGS

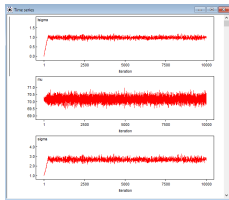
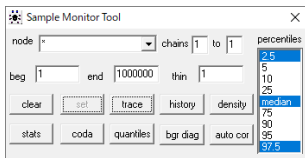
13. Model menu → Update...



14. Update e.g. 10,000 for MCMC iterations and click on **update**



15. Click on **history** in Sample Monitor Tool for the traceplot

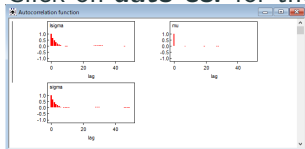


16. Click on **stats** in Sample Monitor Tool for the summary statistics

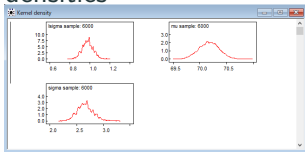
Node statistics

i	node	mean	sd	MC error	2.5%	median	97.5%	start	sample
1	sigma	0.9778	0.05588	0.001588	0.8712	0.9771	1.089	4001	6000
2	mu	70.17	0.1918	0.002717	69.8	70.18	70.54	4001	6000
3	sigma	2.663	0.1495	0.004212	2.39	2.657	2.97	4001	6000

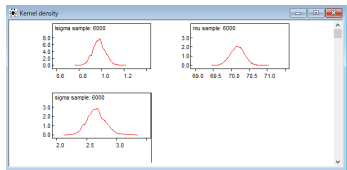
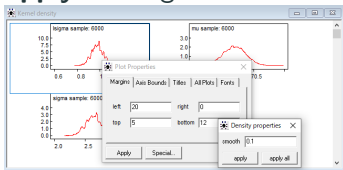
17. Click on **auto cor** for the autocorrelations



18. Click on **density** in Sample Monitor Tool for the posterior densities



19. Right click on one of the figures and choose **Properties...**
20. Click on **Special** and enter **0.1** in **smooth** box. Click on **apply all** to get smooth curves.



21. Click on **quantiles** in Sample Monitor Tool for the traceplot of sample quantiles
22. Click on **coda** in Sample Monitor Tool for the MCMC samples